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On Asset Prices in Exchange and Production Economies

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WP#1914-87 Revised March 1987

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Abstract

This note presents a formal proof that introducing production and capital accumulation in a pure exchange Arrow-Debreu economy will not increase the set of joint equilibrium processes on consumption and asset prices. A direct empirical implication is that if a model is rejected in an exchange paradigm introducing production will not change the result.

This note presents a simple proof to demonstrate that expanding the set of technologies in a pure exchange, Arrow-Debreu economy to admit capital accumulation and production does not increase the set of joint equilibrium processes on consumption and asset prices.

Modifying the technology in an exchange economy to incorporate production introduces restrictions which must be satisfied simultaneously with the equilibrium asset pricing relation such as the one in Lucas (1978).

Intuitively, the equilibrium stochastic process generated by any production economy can also be generated by an exchange economy that is endowed with the same stochastic process.

A direct empirical implication is that if a model is rejected in an exchange paradigm, introducing production and the concomitant restrictions will not change the result.[1] We formalize this below.

Let θ denote preferences, τ technologies, E the set of exogenous processes on the aggregate consumption good, P the set of technologies with production opportunities, and $m(\theta,\tau)$ the set of equilibria for economy (θ,τ).

Theorem

$$\bigcup_{\tau\in E} m(\theta,\tau) \supset \bigcup_{\tau\in P} m(\theta,\tau)$$

Proof. For $\theta_0 \in \theta$ and $\tau_0 \in P$ let (a_0,c_0) be a joint equilibrium process on asset prices and consumption. A necessary condition for equilibrium is that the asset prices a_0 be consistent with c_0, the optimal consumption for the household with preferences θ_0. Thus, if (a_0,c_0) is an equilibrium then

$$a_0 = g(c_0,\theta),$$

where g is defined by the first-order necessary conditions for household maximization. This functional relation must hold for all equilibria, regardless of whether they are for a pure exchange or a production economy.

Let (a_0,c_0) be an equilibrium for some economy (θ_0,τ_0) with $\tau_0 \in P$. Consider the pure exchange economy with $\theta_1 = \theta_0$ and $\tau_1 = c_0$. Our contention is that (a_0,c_0) is a joint equilibrium process for asset prices and consumption for the pure exchange economy (θ_1,τ_1). For all pure exchange economies, the equilibrium consumption process is τ, so $c_1 = \tau_1 = c_0$, given more is preferred to less. If c_0 is the equilibrium process, the corresponding asset price must be $g(c_0,\theta_1)$. But $\theta_1 = \theta_0$ so $g(c_0,\theta_1) = g(c_0,\theta_0) = a_0$. Hence a_0 is the equilibrium for pure exchange economy (θ_1,τ_1), proving the theorem. Since the set of equilibria in a production company is a subset of those in an exchange economy, it follows immediately that the equity premium in an exchange economy will not increase by modifying the technology to incorporate production.

Footnote

1. A related observation is made in Mehra and Prescott (1985) where they assert that incorporating production in their analysis along the lines of Brock (1982) or Donaldson and Mehra (1984) will not increase the equity premium.

References

Brock, W.A., "Asset Prices in a Production Economy", in The Economics of Information and Uncertainty, J.J. McCall (ed.), Chicago: University of Chicago Press (1982).

Donaldson, J.B. and R. Mehra, "Comparative Dynamics of an Equilibrium Intertemporal Asset Pricing Model", The Review of Economic Studies 51 (July 1984): 491-508.

Lucas, R.E. Jr., "Asset Prices in an Exchange Economy" Econometrica 46 (November 1978). 1429-1445.

Mehra, R. and E.C. Prescott, "The Equity Premium: A Puzzle", Journal of Monetary Economics 15 (March 1985): 145-161.

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